Chapter -V Solid State

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Chapter -V **Solid State**

Introduction:

We know that matter generally exist in three states namely; solid, liquid and gas. When we change the physical conditions by altering the pressure and temperature of the surroundings, we find the states to change and transform from one form to another. Solid state is a state of matter besides liquid and gaseous state. In case of solids the intermolecular forces are very strong and empty spaces between the atoms/ions/molecules is very less. That is why they have a fixed shape and volume and high rigidity.

Characteristics of solids:

- 1. Definite mass, shape and volume: Solids have a definite mass, volume and shape because intermolecular forces held the constituent particles of matter together. The intermolecular force tends to dominate he thermal energy at low temperature and the solids stay in the fixed state. In a solid and liquid, the mass and the volume remains the same but solids also have fixed shape unlike liquids. Solid materials tightly packed atoms ensure that a definite shape is maintained.
 - Example: Ice cube remains in the solid state at low temperature, but we find the ice to either melt or evaporate away when we change the physical conditions.
- 2. High density and low compressibility: This is due to close packing of molecules which eliminates free space between molecules.
- 3. Very slow diffusion: Negligible or very slow diffusion as particles have permanent position from which they do not move easily.
- **4. Vapor Pressure:** V.P. of solids is generally much less than V.P. of liquids at definite temperature.
- **5.** Solids are rigid and hard. This is due to lack of space between the constituent particles which make it rigid or fixed.
- **6.** The intermolecular distance between molecules is short. Due to this, the force between the constituent particles (atoms, molecules or ions) is very strong.
- 7. The constituent's particles can only oscillate about their mean positions.
- **8.** Solids are heavier than liquids.(Exception: Ice)

- 9. Solids having strong cohesive forces (Inter particle force) these are stronger than liquid and gases.
- 10. The molecules in solid are fixed at one point. They are held tightly at a position by intermolecular forces of attraction, hence it cannot be poured.
- 11. Inter particle distance is very short that is Intermolecular distance is very short.
- 12. Particle vibrates in place.
- **13.** Solids have low kinetic energy.
- 14. All pure solids have characteristics M.pt. which depends upon intermolecular forces of attraction. It is above the room temperature at the atmospheric pressure.
- **15.** Solids have high density because they have fixed volume.
- **16. Thermal Energy:** Thermal energy is a kinetic form of energy. It is an objects internal energy responsible for its temperature. Thermal energy transfer occurs through heat transfer; with rise in the thermal energy the particles of matter tends to move faster and vice versa with rising temperature thermal energy increasers and teen to move the particles faster.

Crystallography: It is science of crystals which is devoted to the study of crystal development and growth, their external form, internal structure and physical properties. For long time crystallography was regarded as mineralogy. Crystalline substances are very widespread; majority of bodies/objects surrounding us have crystalline structure.

Abbe R. J. Haiiy (1784): Studied regularity of the crystal growth and observed that crystal is made up of very large identical units called as space lattice; it describes three dimensional arrangements of atoms, molecules or ions in crystal lattice.

Classification of Solids: Solids are classified on the basis of following parameters.

- 1. Based on various properties
- 2. Based on bonding present in building blocks

On the basis of various properties solids can be classified as,

- 1. Crystalline solid
- 2. Amorphous solid

Crystalline solid: 1. They has regular and well defined ordered arrangement of particles.

- 2. Crystalline solids have same geometry and regularity of crystal lattice.
- 3. Atoms are closely packed. 4. Examples are KCl, NaCl, ice etc.

Amorphous solid: 1. They has no regular and well defined ordered arrangement of particles.

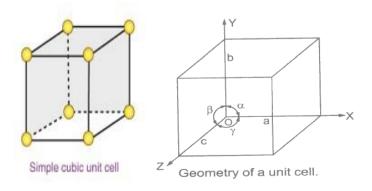
- 2. Amorphous solid does not have same geometry and regularity of crystal lattice.
- 3. Examples are Rubber, amorphous sulphur etc. glasses are sometime called amorphous solids and sometimes super cooled liquids of high viscosity.

Difference between Crystalline and Amorphous solid:

Sr.No.	Property	Crystalline solid	Amorphous solid
01	Shape	Have Long range order	Have short range order
02	Melting point	Have definite Melting point	Do not have definite Melting point
03	Heat of fusion	They have definite heat of fusion.	Do not have definite heat of fusion.
04	Compressibility	They are rigid and incompressible	May be compressed to some extent.
05	Cooling with sharp edged tool	It breaks up into smaller crystals of same geometrical shape.	Broken pieces are not generally flats/have irregular surface.
06	Symmetry	They possess symmetry.	They do not possess any symmetry.
07	Volume change	Sudden change in volume when they melt.	No Sudden change in volume on melting.
08	Interfacial angles	They possess Interfacial angles.	They do not possess Interfacial angles.
09	Cooling Curve	Cooling curve is smooth.	Cooling curve has two breaks ('a' and 'b') which correspond to beginning and end of process of crystallization.

Sr.No.	Property	Crystalline solid	Amorphous solid
10	Isotropy and Anisotropy	They are anisotropic (Physical	They are Isotropic (Physical
		properties are different in	properties are same in all
		different directions)	directions)
		Anisotropy in crystalline of particles [crystalline solid]	(b) Isotropic uniform structure
		Anisotropic Isotropic Anisotropic	Anisotropy in Crystals Isotropy in Amerybous solids
11	Examples	NaCl, Quartz, Diamond,	Silicon, Plastics, glass, rubber,
		Sodium Nitrate,	polymer, gel, fused silica, thin
		Sugar,Snowflakes,CaF ₂ ,	film lubricant etc.
		SiO ₂ ,Alum etc.	

Unit cell: The smallest repeating unit of the crystal lattice is the unit cell, the building block of crystal. Or the smallest unit of the arrangement of atoms in a crystal is known as unit cell. **Example:** Crystal of calcite (CaCO₃) can be broken into large number of small rhombus which has original shape; even if it again and again broken, a point would be reached at which the smallest crystal would exists.

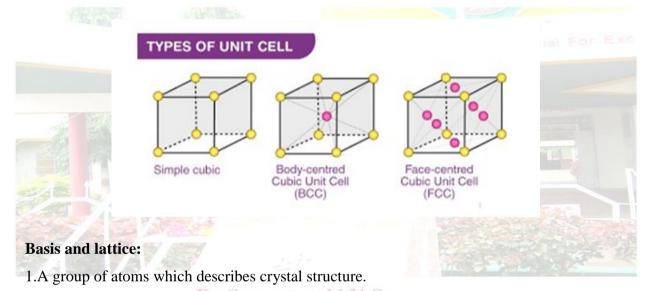


Characteristics of Unit cell:

- 1. a, b and c are lengths of the edges of unit cell.
- 2. α , β and γ are angles between three imaginary axes.
- 3. OX, OY and OZ are three imaginary axes.
- 4. Unit cell has **8** corners & at each corner eight cells meets.

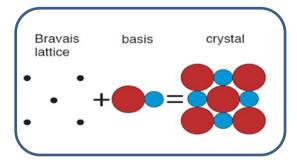
There are three types of unit cell:

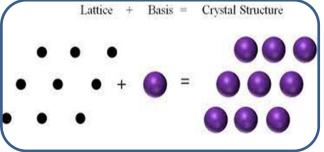
- **1.Simple unit cell:** Atoms, ions or molecules are present only at the corner of the unit cell.
- **2. Face centered:** Besides the corner, there is a point present in the center of each face.
- **3. Body centered:** There is point at the center within the body of the unit cell.



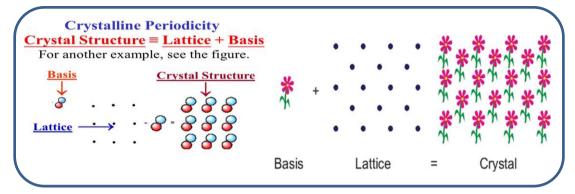
- 2. A lattice is a hypothetical regular and periodic arrangement of points in space.
- 3. A basis is a group/collection of identical atoms/molecules in particular fixed arrangement.

We could have a basis of a single atom as well as a basis of a complicated but fixed arrangement of hundreds of atoms.



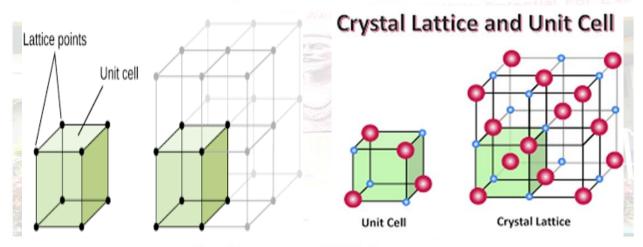


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Crystal lattice: The unit cells which are all identical are defined in such a way that they fill space without overlapping. The 3D arrangement of atoms, molecules or ions inside a crystal is called a *crystal lattice or space lattice*. *Crystal lattice* is made up of numerous unit cells.

Lattice point: A lattice point is a point at the intersection of two or more grid lines in a regularly spaced array of points, which is a point lattice.



Weiss indices and Miller indices: Indices is nothing but Indicator of crystal planes. The Weiss parameters, introduced by Christian Samuel Weiss in 1817. It will help us to determine the distance between the parallel planes of the crystal or it characterizes and represents any plane of the crystal. Weiss indices are ratio of intercept on that axis to the unit length on that axis.

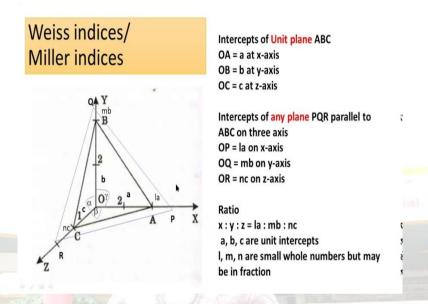
Weiss indices = Intercept on the axis / Unit length on the same axis. Weiss indices are generally small whole number and may be fractions of whole numbers as well as infinity.

Conditions for Weiss indices:

1. Three axes present namely a, b, c values (Weiss indices value) may be fraction, integrals, positive, negative, infinite, but never be zero.

2. Conditions for positive and negative in Weiss indices (W.I.):

- 2.1 If origin to right then W.I.is positive and origin to left W.I. will be negative.
- 2.2 If origin to front then W.I.is positive and origin to back W.I. will be negative.
- 2.3 If origin to up then W.I.is positive and origin to down W.I. will be negative. Origin cannot be present along with the plane.



Miller indices: (William Hallowes Miller 1839)

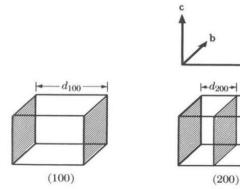
- 1. It is mathematical notation to represent crystal plane and its direction in crystal structure.
- 2. Miller indices of a plane are the reciprocals of the distances from the origin at which a given face intersects the three axes.
- 3. In Miller indices (M.I.) h, k, l is used for different axis instead of 'x', 'y' and 'z' axis. That is 'x' is replaced by 'h', 'y' is replaced by 'k' and 'z' is replaced by 'l
- 4. M.I. obtained by taking the reciprocals of Weiss coefficients and multiplying by the smallest number that will express all the reciprocals as integers.

$$h = \frac{a}{Intercept \ of \ plane \ along \ 'x'axis}$$

$$h = \frac{b}{Intercept \ of \ plane \ along \ 'y'axis}$$

$$h = \frac{c}{Intercept \ of \ plane \ along \ 'z'axis}$$

- 5. Larger the value of M.I., smaller will be the value of the intercept of that plane along the given axis when it is zero (The intercept is infinity), plane is parallel to that axis.
- 6. Two parallel planes in crystals have same miller indices, because they are equally spaced parallel planes.



How to calculate M.I?

Selection of origin: Rule-I: As per definition of origin, origin should be left, down and back. **Rule II:** Origin can't be present along with the plane x and y cannot taken as origin. Find the intercept on the respective axis (h, k, l).

- 1. Obtain the reciprocal of Weiss indices.
 - 2. Eliminate on of fractions to make them integrals (Whole numbers). Miller indices of the plane are (h, k, l).
 - 3. Origin is never considered on the plane because if considered intercept will become zero and thus M.I. will become infinite but we know that M.I. is never infinite.
 - 4. Origin is considered /fixed on such a point for which point in one step we should reach /move to it from plane. Or in other words we may say that distance of plane from origin should be one unit.

General Principles for M.I:

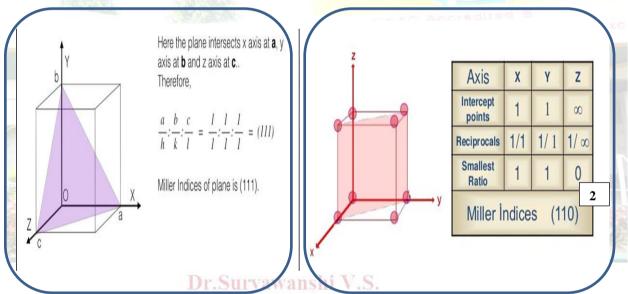
- ➤ If a Miller index is zero, the plane is parallel to that axis.
- The smaller a Miller index, the more nearly parallel the plane is to the axis.
- ➤ The larger a Miller index, the more nearly perpendicular a plane is to that axis.

- ➤ Multiplying or dividing a Miller index by a constant has no effect on the orientation of the plane.
- ➤ Miller indices are almost always small.

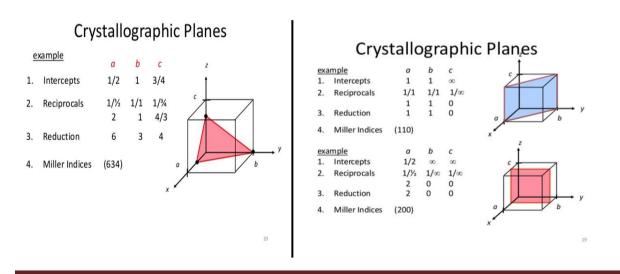
Rules for Miller Indices:

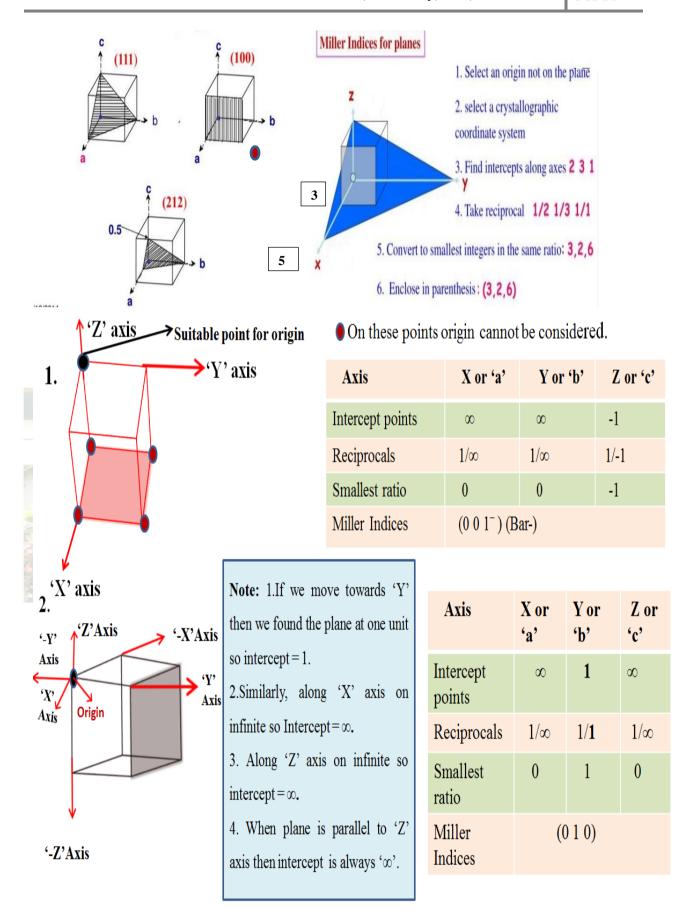
- 1. **Determine the intercepts** of the face along the crystallographic axes, in terms of unit cell dimensions.
- 2. Take the reciprocals.
- 3. Clear fractions.
- 4. Reduce to lowest terms.

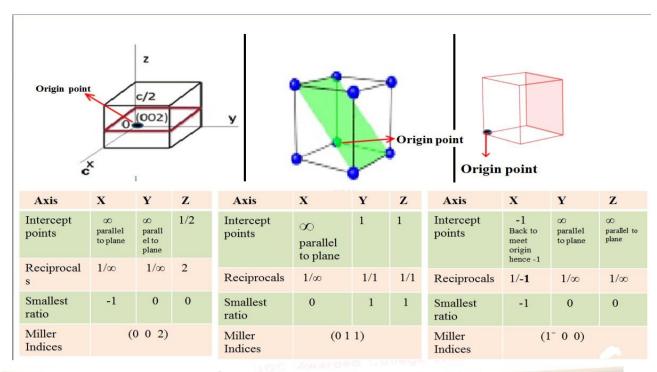
Conditions for Miller indices: It can be integer, positive, negative or zero but never be infinite or fraction. If negative then it is denoted by 1⁻ (Bar-).

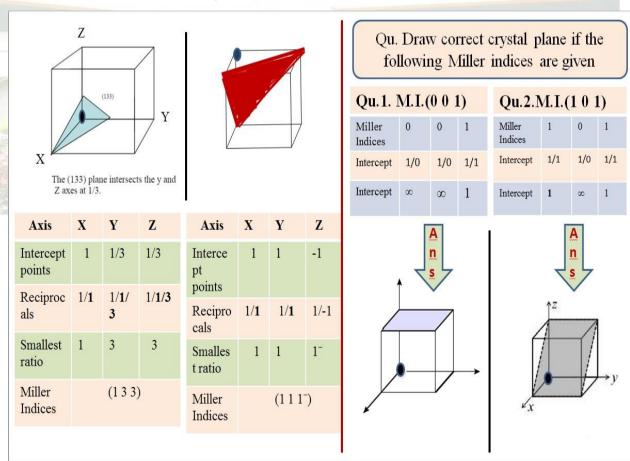


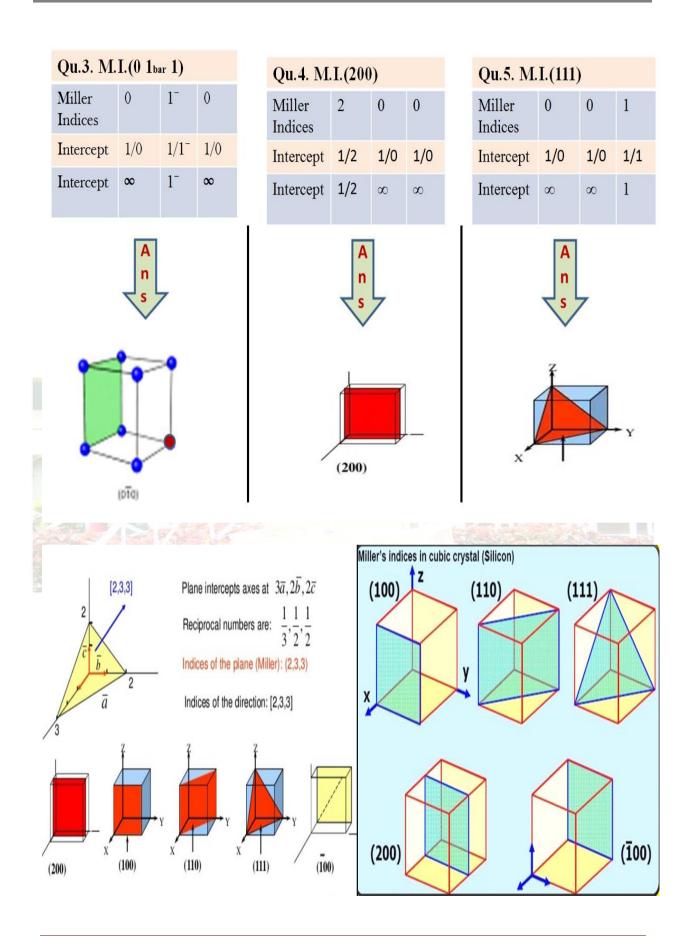
More examples: China and C

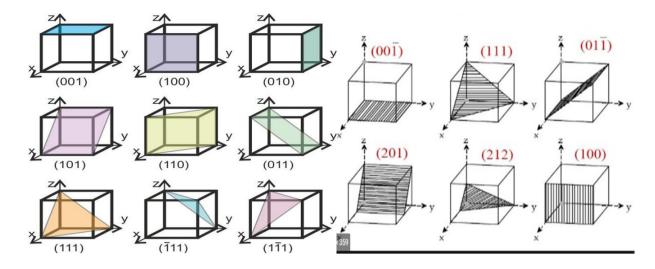












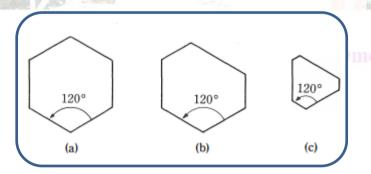
Laws of Crystallography: The geometric crystallography is based on three fundamental laws;

1. Law of constancy of interfacial angles: Angle between adjacent corresponding faces is interfacial angles of the crystal of a particular substance is always constant in spite of different shapes and sizes and mode of growth of crystal. The size and shape of crystal depend upon the conditions of crystallization. This law is also called as First law of crystallography.

OR

The crystals of the same substance can have different shapes, it depends on number and sizes of faces but the angle between the corresponding faces remains always equal.

In short, that the size and shape of a crystal of a compound varies with conditions of crystallization, but the angle between the faces (Interfacial angle) remains constant.



2. Law of Symmetry: It states that, all crystals of the same substance possess the same elements of symmetry.

Symmetry in the crystals may be due to a plane, a line or a point, on the basis of this law. There are three types of symmetry.

- 1. Plane of symmetry
- 2. Axis of symmetry
- 3.Centre of symmetry.

3. Law of rational indices (Havy law): The ratio between the intercepts on the axes for the different faces of the crystal can be expressed by rational numbers.

Example: la: mb: nc where l,m and n are small whole numbers and a,b and c are the intercepts made by unit plane along the crystallographic axes.

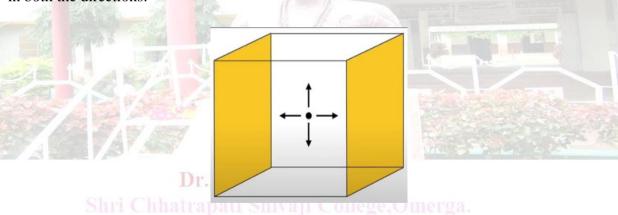
Elements of symmetry: The large number of crystals is formed by different substances, depending on the condition of crystallization, each substance having a characteristics crystalline form(s).

There are more than 200 crystal forms, so that there is problem to classify them, but on the basis of elements of symmetry they can be grouped into small number as different classes.

- 1. Centre of symmetry
- 2. Plane of symmetry
- 3. Axis of symmetry

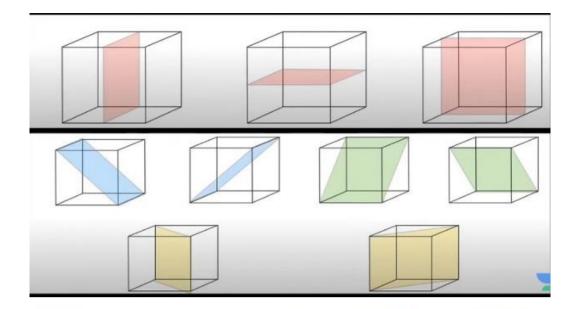
Centre of symmetry: It is defined as an imaginary point within the crystal such that any line passing through this point intersects the opposite face of the crystal at equal distances.

In other words, any line drawn through this point will intersect surface at equal distance in both the directions.



Plane of symmetry: It is an imaginary plane which divides the crystal into two equal parts such that one is the mirror image of the other.

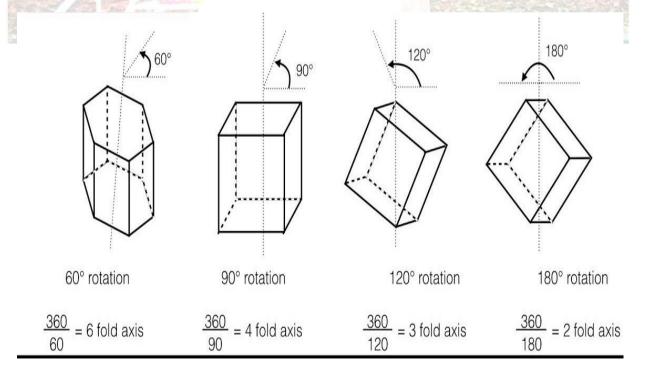
For example Regular cubic structure can have 9 planes of symmetry. Three planes of symmetry are parallel to the cube faces and six diagonal planes are passing through opposite cube edges.

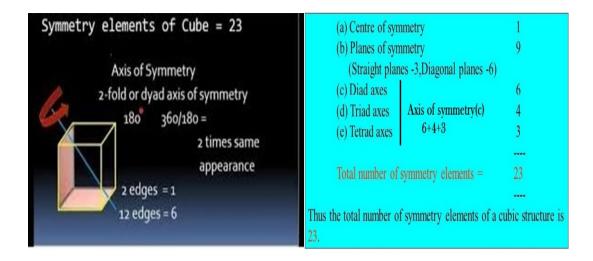


Axis of symmetry: Axis of symmetry is a line about which the crystal may be rotated such that it presents the same appearance more than once during the complete revolution through 360θ

If a crystal presents the same appearance 'n' times in complete revolution, the axis is said to be 'n' fold symmetry. The cube contains three different types of symmetry axes.

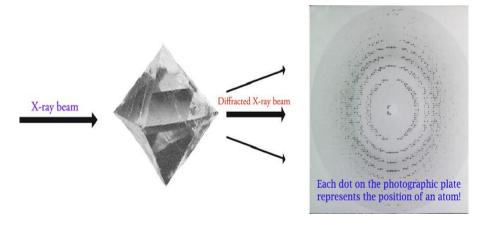
- A) Three-4 fold axes, each of which passes through the centers of two opposite faces.
- B) Four-3 fold axes, each of which passes through two opposite vertices and
- C) Six-2 fold axes, such of which passes through the midpoints of two opposite edges.





X-ray diffraction from crystals:

- X-ray diffraction or X-ray crystallography is used to determine crystal structures/atomic and molecular structure by interpreting the diffraction patterns formed when x-rays are scattered (diffraction in many specific directions) by the electrons of atoms in crystalline solids.
- X-ray are sent through a crystal to reveal the pattern in which the molecules and atoms.
- > X-ray crystallography works based on X-ray diffraction principle; it means that scattering of x-ray by crystal.
- X-ray diffraction is based on constructive interference of monochromatic x-rays and crystalline sample.
- Fivery crystalline substance gives a patterns; the same substance always gives the same pattern; and in a mixture of substances each produces its pattern independently of the others"



Laue's photographic method:

- ➤ Max Von Laue (1912) passed continuous beam of X-rays with different wavelength through a crystal to get a diffraction pattern to get positions of ions or molecules in crystals.
- ➤ The reason for use of X-ray that, the wavelength of X-ray was of about the same order as the interatomic distances in a crystal. X-ray penetrates more than the cathode rays.
- > X- Ray has High frequency $(3x10^{16} \text{ Hz to } 3x10^{19} \text{ Hz})$ and high energy but low wavelength $(10^{-8} \text{ to } 10^{-11} \text{m})$ than visible light.
- ➤ Electric and magnetic field have no influence on X-rays and so they evidently do not consist of charged particles.
- The fine beam coming out of the slit (Lead) passed through crystal placed on the holder.
- The beam emerging from the crystal was made to fall on a photographic plate, kept few Cm from crystals.
- Photograph is obtained, it consists of a central spot which arises due to the un-diffracted beam, and the central spot is surrounded by spots which arise from different diffracted beam. Thus pattern of spots gives idea about the crystal structure.
- The math's involved in this method for calculations are highly complex.

Diffraction:

The process by which a beam of light or other system of waves is spread out or bending of EMR as a result of passing through a narrow aperture or across an edge, typically accompanied by interference between the waves forms produced. Dark band observes due to constructive interference.

Diffraction takes place with electromagnetic radiation like light, X-ray and gamma rays and with very small particles such as atoms, neutrons and electrons which show wave like properties.

Interference of light:

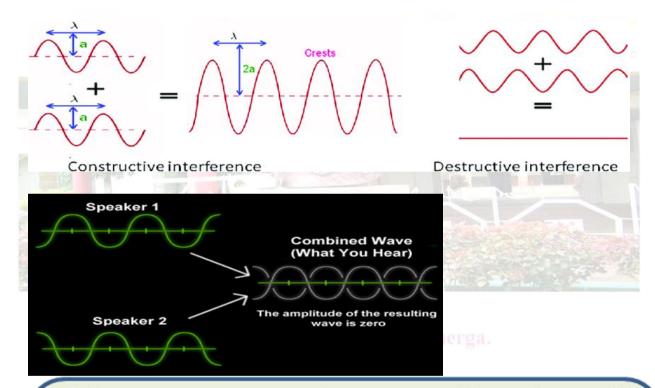
- It is the phenomenon in which two waves superpose to form the resultant wave of lower, higher or same amplitude.
- The most commonly seen interference is the optical interference or light interference.

1. Constructive interference: It occurs when the maxima or minima of two waves add together (the two waves are in phase), so that the amplitude of the resulting wave is equal to the sum of the individual amplitudes. (higher amplitude).

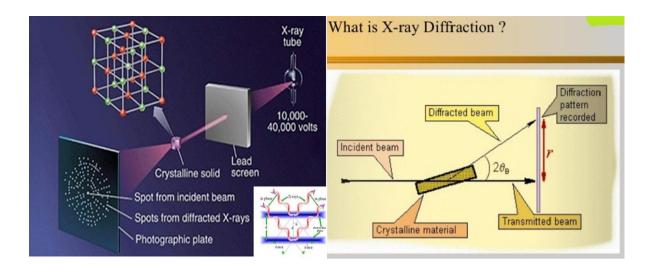
For example: Two speakers playing same music while facing each other. At this time, music will appear louder and powerful as compared to music played by single speaker.

2. Destructive interference: When the maxima of two waves are 180 degrees out of phase: a positive displacement of one wave is cancelled exactly by a negative displacement of the other wave. The amplitude of the resulting wave is zero.

The dark regions occur whenever the waves destructively interfere.

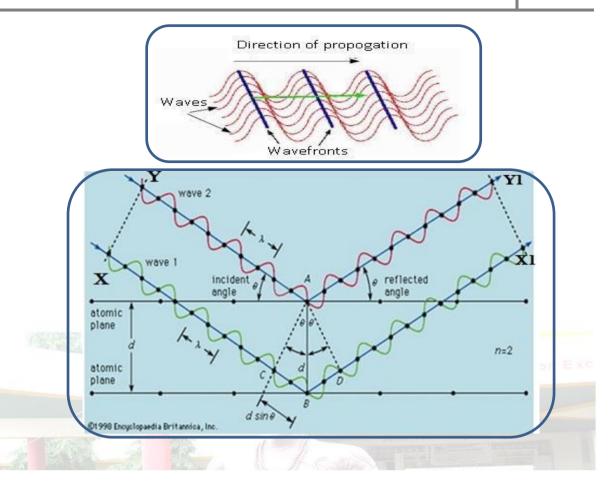


If the **speakers** are at the same position, there will be constructive **interference** at all points directly in front of the **speaker**. If the **speakers** are separated by half a wavelength, then there is **destructive interference**, regardless of how far or close you are to the **speakers**.



Bragg's Equation: W.Henry and W.L.Bragg devised and studied that each crystal plane reflects radiation as a mirror and analyzed this situation for cases of constructive and destructive interference.

- When a beam of X-rays is passed through a crystal each atom in the path acts as scattering center and thus emits secondary radiations.
- However, reflection of X-rays can take place at certain angles when the path difference of emitted ray from the successive planes is a whole number multiple of the wavelength.
- Consider a set of parallel atomic planes whose spacing is 'd' and let a narrow monochromatic beam of X-rays fall upon these parallel planes at a glancing angle ' θ '
- Each atom in a given layer, according to Huygens's principle, becomes the center of expanding wavelets whose envelope gives rise to reflected wave front.
- As in the case of optical reflection, the reflected wave front will have maximum intensity at angle ' θ ' to this plane, which is equal to the glancing angle.
- \triangleright Each parallel layer/planes (third, fourth, fifth etc.) in the given set gives rise to the reflected wave front (for I and II, X_1Y_1).
- **Condition for constructive interference:** Path difference between reflected wave fronts from one layer and that from the next must be equal to an exact wavelength or an integral multiple of wavelength $(n \ \lambda)$, where 'n' is an integer and ' λ ' is wavelength.
- ➤ Wave fronts: Time-varying field is the set (locus) of all points where the wave has the same phase.



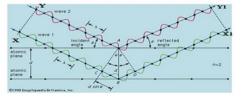
- ➤ Each crystal diffracts to X-ray because interplanary spacing in a crystal lattice is of the same order as that of wavelength of X-rays.(X-rays can penetrates to crystal lattice).
- Draw a perpendicular line from A to B, similarly draw AC and AD perpendicular to the direction of the incident and reflected rays.
- The rays reflected from B travels a longer distance than that reflected from A.(path-II > path-I).
- \triangleright Each of these lines makes an angle θ with AB whose length is equal to the spacing d.
- Path difference = BC + BD (path diff.= BC + BD) should be equal to $(n.\lambda)$ (1) that is, BC+BD=n λ

$$BC + BD = n.\lambda$$
 or path difference= $n.\lambda$ (2)

In triangle ACB, $\sin\theta = \frac{BC}{AB}$

It may written as, $\frac{BC}{d \ (diagonal)}$ Or $\boxed{BC = d \sin \theta}$

Similarly in triangle ABD, $\sin \theta = \frac{BD}{AB}$ or it may be written as, $\frac{BD}{d \ (diagonal)}$ or $\boxed{BD = d \sin \theta}$



Path difference = BC + BD =
$$d \sin \theta + d \sin \theta$$

= BC + BD = $2 d \sin \theta$ (3)

Path difference = BC + BD =
$$d \sin \theta + d \sin \theta$$

= BC + BD = $2 d \sin \theta$ (3) Note that :

Equating equation 1 and 3 we get, $n.\lambda = 2d \sin \theta$ or $d = \frac{n.\lambda}{2 \sin \theta}$

Where,

 θ - is angle on which x-rays falls.

d - is interplanar separation.

n - is order of refraction (integer 1, 2, 3, 4.... & so on)

Braggs Law:

Constructive interference of the radiation from successive planes occurs when the path difference is an integral number of wavelength.

d, n, λ and θ are variables, if any three can known we calculate forth.

Problem: Calculate the distance between two layers/scattering planes if n=2 (Second order), λ = 1 Angstrom, θ is 60 degree ?

Ans: We know that, $d = \frac{n \lambda}{2 \sin \theta} = \frac{2 X1}{2 \sin \theta} = \frac{2}{2 \sin \theta} = \frac{2}{2 \sin \theta} = 1.15 \text{ AU}$ is the distance between two layers of crystal.

Applications of XRD / Why XRD?

- 1. By measuring the angles and intensities of these diffracted beams, a crystallographer can produce a 3D of electron density within the crystal.
- 2. Measures the average spacing between layers or rows of atoms.
- 3. Determines the orientation of a single crystal or grain.
- 4. Finds the crystal structure of an unknown material.
- 5. Measures the size, shape and internal stress of small crystalline regions.
- 6. Electron density and accordingly position of the atoms in complex structures (position of atoms, chemical bonds, their disorder etc. can be determined.
- 7. Useful to determine whether the compound is solvated or not.
- 8. Useful to determine cis-trans isomerization and state of anneal in metals.
- 9. It is novel and nondestructive method of chemical analysis.